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Self-Similar Cosmological Model: Introduction and Empirical Tests

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After calling attention to the empirical and theoretical motivations for considering the hypothesis of a self-similar cosmos, the basic concepts and scaling rules of the Self-Similar Cosmological Model are presented. The results of a diverse set of 20 falsification tests are then shown to provide strong quantitative support for the uniqueness and broad applicability of the self-similar scale transformation equations, which successfully correlate physical parameters of atomic, stellar, and galactic scale systems. Possible implications of these results are discussed.

... the wise man looks into space
and does not regard the small as too little,
nor the great as too big,
for he knows that there is no limit to dimensions.
Lao-tse

1. INTRODUCTION

1.1. Goals of the Review

The Self-Similar Cosmological Model (hereafter SSCM) is a heuristic cosmological model that has been developed over the past 10 years in a series of 17 papers by the author (Oldershaw, 1978–1987*b*, 1989*a*,*b*). The major goal of this review is to introduce the SSCM, its 20 successful falsification tests, and its major predictions to as large and diverse an audience of scientists as possible. Other goals of this review are a reasonably compact summary of the previous work on the SSCM and the identification and clarification of various modifications to the model that have occurred over the past 10 years. The remainder of this section will be concerned with the question of why one should be interested in an unorthodox self-similar model of the cosmos. Section 2 will introduce the SSCM in its simplest and

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most general form and Section 3 will discuss the substantial amount of empirical evidence in favor of cosmological self-similarity. The sequel (Oldershaw, 1989*c*) to this paper will present a more detailed and technical discussion of the SSCM, including its major predictions, implications, and unresolved problems.

1.2. Reasons for Considering a Self-Similar Cosmological Model

1. The overwhelming majority of physicists currently think that the Big Bang cosmology (augmented by Inflation) and the Standard Model of particle physics (and subsequent unification theories) are unquestionably the right theories to guide us toward a fully unified understanding of nature; some even predict that all fundamental questions in physics will be solved in the near future by pursuing these theoretical paths. On the other hand, even supporters of these theories admit that their theoretical constructs are often untestable in a definitive way, that they have had trouble with most of the few falsification tests that have been identified, and that they have been unable to anticipate major new observational discoveries. This situation has been detailed elsewhere (Oldershaw, 1988) and will not be repeated here in full, but let us briefly consider the current state of affairs in cosmology. The Big Bang theory has always encountered serious theoretical problems, such as the flatness problem, the smoothness problem, and the horizon problem. These problems were "solved" by the *ad hoc* addition of an Inflationary episode at about 10^{-35} sec after the Big Bang, but this solution leads to other equally serious problems. For example, the major prediction of the Inflated Big Bang theory is that the matter density of the universe equals the critical density (i.e., $\Omega = 1$), but this prediction has been contradicted by most observationally based estimates made to date (Rothman and Ellis, 1987). This theory also leads to potential conflicts between the predicted age of the universe and the estimated ages of its oldest constituents (Tayler, 1986). Moreover, the Inflation scenario is totally dependent upon the validity of the GUTs of particle physics, which are themselves beset by falsifications, arbitrariness, and testability problems (Pickering, 1984). Even more worrisome is the fact that the Big Bang theory failed to anticipate major empirical discoveries of recent years, such as the large-scale inhomogeneity in the distribution of matter, the large deviations from a smooth Hubble flow and, most importantly, the dark matter constituting more than 90% of the matter of the universe (Oldershaw, 1988). None of the variations on the Big Bang theme can provide a convincing explanation for the existence of galaxies, and the Hubble constant is uncertain by a factor of 2. In short, there is no justification for complacency with regard to our current state of knowledge in the field of cosmology (or particle physics).

2. For the past 10 years there has been a growing interest in the "fractal" properties of nature's geometry, largely due to the inspiration of Mandelbrot (1982). Fractal structures usually involve self-similarity, a form of invariance with respect to transformations in scale, in which small parts of a structure have geometrical properties that resemble the whole structure or larger parts of the structure. A Russian doll of the type that has a doll within a doll within a doll is a clearcut example of a self-similar structure. Mandelbrot and many others who have followed his lead have identified examples of self-similarity everywhere in nature: the clustering of galaxies, stars, or atomic particles in a plasma; the branching of trees, rivers, or circulatory systems; the cratering of astronomical bodies; the patterns of crystal growth; the motions of turbulent fluids; the shapes of coastlines; the topology of mountain ranges; etc. In fact it is difficult to think of any realm of nature that does not include nontrivial examples of self-similarity. Although we do not as yet have a fully satisfactory explanation for why self-similarity should be so ubiquitous, we can unequivocally say that self-similarity is one of nature's fundamental design properties. It seems reasonable and natural to suspect that the solution to nature's biggest design problem, the design of the cosmos itself, might involve self-similarity.

3. When a theory or paradigm is regarded as possessing beauty or elegance, terms often associated with Einstein's General Theory of Relativity or Darwin's Theory of Evolution, for example, it is meant that the theory is *conceptually simple* and permits a pleasing *unification* of previously disjoint facts or ideas about nature. As I hope to demonstrate in this paper, the SSCM is conceptually very simple and proposes an unprecedented degree of unification in the physics taking place on all scales of nature's hierarchy. Therefore, the SSCM has the potential for being a remarkably beautiful theory; one scientist (Sagan, 1980) has referred to the general notion of an infinite hierarchical cosmos of self-similar systems as "one of the most exquisite conjectures in science or religion." These potential attributes do not constitute rigorous scientific support for the SSCM, but they do argue that it deserves serious, open-minded consideration.

4. Another reason for exploring the possibility of a self-similar cosmos is that it avoids at least three major philosophical problems that have raised concerns about modern physics. First, the Big Bang theory proposes that the "initial state" of the entire universe was that of a singularity with a radius of zero (space and time did not exist yet), but with infinite pressure, temperature, and density. One might well ask how these latter quantities could have any meaning without space-time. Moreover, the hypothetical initiation of the expansion of the universe from a singular state represents an unexplained, acausal event, a fact that is often downplayed in discussions of the Big Bang theory, but which is obviously a theoretical drawback. The

SSCM avoids this problem by interpreting the large-scale expansion that we now observe as a *local* phenomenon taking place in one particular metagalactic system on the metagalactic scale of the hierarchy, much as stellar and galactic scale systems can explode or undergo rapid expansion from a more compact state.

Second, the Big Bang theory makes the extremely suspect assumption that nature's hierarchy ends at about the scales where our observational capabilities end, and that we just happen to find ourselves in the vicinity of the center of that scale range. In the SSCM, on the other hand, there is no spectre of an anthropocentric truncation of nature's hierarchy, since it postulates that the hierarchy extends well beyond current observational limits, and is perhaps completely unbounded.

Finally, modern physics has something of a split personality in that the physics of the microworld is hypothesized to be inherently different from the physics of the macroworld, with a somewhat fuzzy interface between these two realms wherein quantum microphysics rather mysteriously metamorphoses into classical macrophysics. The SSCM hypothesizes that one set of physical laws holds good for all scales of nature's hierarchy.

Therefore, the fact that the SSCM is not plagued by these three philosophical problems is another reason for giving it due consideration.

5. The strongest argument for studying the concept of a self-similar cosmos is the considerable amount of quantitative evidence that supports it. In Section 3, 20 successful tests of the SSCM are discussed, and it is shown that the match between theoretical predictions and observational estimates far exceeds that expected by chance, or even that which could be achieved by numerical "fudging." Compare this degree of empirical support with that achieved by the highly regarded GUTs of particle physics (Oldershaw, 1988).

For these five reasons, then, it would appear that the SSCM is worthy of serious attention in spite of its divergence from generally accepted cosmological assumptions.

2. GENERAL DISCUSSION OF THE SELF-SIMILAR COSMOLOGICAL MODEL

2.1. Heuristic Status

It should be understood from the outset that the SSCM is still very much in the heuristic stages of development. That is, the SSCM and the properties of nature which led to the formation of its major hypotheses can be described in some detail and the self-similar scaling equations that are the heart of the model can be empirically derived and quantitatively tested, but the SSCM cannot as yet answer the basic questions of why nature has a self-similar design and why the two dimensionless constants of the scaling equations have the particular values that are found empirically. The heuristic status of the SSCM may be viewed as a shortcoming, but on the other hand it would seem to be unwise to risk stunting the development of the SSCM by encasing the promising heuristic core in a hastily constructed theoretical shell.

Because the SSCM proposes a fundamentally different understanding of nature, and because it would therefore significantly alter ideas in most branches of theoretical physics, it is of considerable importance when studying this model that: (1) all previous *theoretical* constructs should be regarded as being open to question (a basic tenet of science), and (2) that observational data should take primacy over theoretical assumptions or expectations when conflicts between these two occur. Unfortunately, at present this is not always the case (Pickering, 1984).

2.2. Hierarchical Organization of the Cosmos

It is a self-evident fact that nature has a nested hierarchical organization, though this fact is often taken too much for granted. In our local planetary environment, for example, "elementary particles" combine to form atoms, which are the building blocks of molecules, which compose a vast array of macroscopic objects, which are collected into planets, moons, asteroids, and comets, which are components of the solar system. Taking a more cosmological perspective [in terms of the ubiquity (Oldershaw, 1985) of the building blocks, the breadth of the spatial domain under consideration, and the range of physical scale] it is known that electrons, atomic nuclei, and ions are the primary building blocks of stars, which are thought to be the primary building blocks of galaxies, which are clustered into ever-larger aggregations until the limits of our observational abilities are approached.

Considering *well-defined* classes of relatively *stable* objects that have *mass*, several major characteristics of the *observable* portion of the cosmological hierarchy can be identified. The important question of the form of the dark matter has been discussed previously (Oldershaw, 1986*a*,*d*) and will be a primary topic of the sequel (Oldershaw, 1989*c*) to this paper.

1. As just mentioned, the cosmological hierarchy has a *nested* organization wherein smaller objects are components of larger objects, which, in turn, are components of even larger objects.

2. If we were to consider each class of objects as defining a "level" in nature's hierarchical arrangement, and order these levels in terms of increasing mass or range of masses associated with each class, then the entire cosmological hierarchy involves a *quasicontinuous* (Oldershaw, 1985) set

of levels ranging in mass from about 10^{-27} g to at least 10^{45} g. Since building blocks at the levels of "elementary particles" appear to have very discrete masses, a hierarchy that includes them as major building blocks cannot be perfectly continuous in terms of the range of masses of its constituents.

3. Although the overall mass range of objects comprising the cosmological hierarchy is quasicontinuous, a relatively few classes of objects account for large percentages of the mass of the observable universe, whereas objects at most other levels account for infinitesimal percentages of the observable mass (Oldershaw, 1986*c*). This means that the cosmological hierarchy is *highly stratified* (Mandelbrot, 1982), and this concept is best illustrated with some observational data.

At least 99.9% of all observable mass is in the form of atomic and subatomic objects representing the relatively narrow mass range of roughly 9×10^{-28} g for the electron to about 9×10^{-23} g for an iron atom. This mass range will tentatively be defined as the atomic scale of the cosmological hierarchy, though the choice of cutoffs is somewhat arbitary (Oldershaw, 1985). Between the latter level and the levels at which stellar scale objects commence (about 10^{29} g) only a cosmologically insignificant amount of total mass is in the form of the objects comprising the levels of this interscale range. But *at least* 90% of the total observed mass is bound up in stellar and substellar objects ranging in mass from about $10^{-4}M_\odot$ to $8M_\odot$ ($M_\odot$ is one solar mass, or about 2×10^{33} g); this mass range will be referred to as the stellar scale. Levels above the stellar scale do not involve appreciable percentages of the cosmological mass until one reaches levels designating galaxies, which range in mass from about $10^7M_\odot$ to $10^{12}M_\odot$. *At least* 95% of all observable matter is bound up in objects populating these levels, which are defined as the galactic scale of the cosmological hierarchy. These three scales, involving about 15 orders of magnitude in mass out of a total of about 72 orders of magnitude, dominate the observable portion of the cosmological hierarchy and in this sense the hierarchy can be regarded as quite stratified. Moreover, there is evidence of further stratification within the atomic and stellar scales. In terms of abundances, approximately 99% of all atoms are accounted for by just two classes of atomic systems: hydrogen (in neutral and H^+ states) and helium (in neutral, He^+, and He^{2+} states). Likewise on the stellar scale just two classes of stars, M dwarf and K dwarf stars, account for about 99% of all observable stellar scale systems, as will be discussed in Section 3. Whether a similar degree of stratification occurs within the galactic scale cannot be determined at present due to uncertainty in galactic mass estimates and limitations related to the available sample of galaxies (see point 5 below). Therefore the degree of stratification characterizing the cosmological hierarchy is rather remarkable and worthy of some reflection. From a cosmological perspective the observed portion of nature is, to a rough first approximation, comprised of galaxies which, when looked at "microscopically," are composed of $(0.1\text{-}0.8)M_{\odot}$ dwarf stars which, when looked at "microscopically," are composed of hydrogen and helium. All of the other *known* classes of objects, most of which populate interscale levels, are very minor components of the cosmological hierarchy in the sense that when you add up the mass incorporated in any of these classes and compare it to the total observed mass of the cosmos, the resulting percentage is always less than 15%, and usually infinitesimal.

4. If it is assumed that the hierarchy has uppermost and lowermost levels, then this is a purely *theoretical* assumption. Historically, whenever observational capabilities have been significantly improved, new levels of the cosmological hierarchy have been revealed. Currently the putative bounds of nature's hierarchy are again tellingly close to the largest and smallest size scales that can be adequately observed, and it would appear to be unwise to regard the matter as being closed. The safest bet is that new levels of nature's hierarchy remain to be discovered.

5. At face value it appears that the degree of stratification might decrease above the galactic scale, and that the cosmological hierarchy is asymmetric, since the atomic and stellar scales are separated by about 51 orders of magnitude in mass, whereas the stellar and galactic scales appear to be separated by only about 7 orders of magnitude in mass. With regard to the stratification question, the sample of observable galaxies is relatively small, about 10^{11} galaxies as compared with about 10^{23} stellar objects and about 10^{80} subatomic particles. Moreover, one must take into account the exceedingly small galactic scale "sampling ratio," defined as the ratio of the radius of the observable portion of the universe to the average radius of the systems of a particular scale. The galactic scale sampling ratio is a paltry 10^5 as compared to 10^{17} for the equivalent stellar scale ratio and an awesome 10^{40} for the equivalent atomic scale ratio. Therefore, it is possible that only a tiny sampling of the objects and phenomena occurring on size scales at and above the galactic scale is currently available for scrutiny and this circumstance is a serious hindrance to an accurate evaluation of the actual stratification above the galactic scale. Concerning the issue of hierarchical asymmetry, the SSCM proposes that the cosmological hierarchy is *symmetric* in all respects and empirical tests presented in Section 3 seem to support the symmetry hypothesis. This controversial issue has been discussed previously (Oldershaw, 1986*a,d*) and will be given further attention in the sequel to this paper (Oldershaw, 1989*c*).

2.3. Discrete Self-Similarity

Given the highly stratified organization of the observable portion of the cosmological hierarchy, it seemed natural to compare atomic, stellar, and galactic scale systems with regard to similarities and/or dissimilarities. Taking into account the huge differences in spatiotemporal scale which tend to obscure inherent similarities to a degree that is often seriously underestimated, I found that there was a considerable potential for physically meaningful analogies among atomic, stellar, and galactic scale systems. Let us consider several general examples that suggest the *possibility* of interesting parallels between the physics operating on different scales.

1. In the most general terms, typical systems from all three scales involve distinct objects orbiting one another under the influence of attractive (usually proportional to $1/r^2$) "forces," e.g., atoms, the solar system, and small galactic groups.

2. Relatively large-scale and highly collimated jets of matter, often in a back-to-back configuration, are observed in both stellar and galactic scale systems. The potential for meaningful physical analogies between these jet phenomena has long been advocated by astrophysicists (Geldzahler *et al.*, 1981; Geldzahler and Fomalont, 1986).

3. Likewise, astrophysicists have noted that in some ways, such as their enormous densities, neutron stars are like stellar scale counterparts to nuclear objects on the atomic scale. Moreover, atomic scale objects can range in size from compact nucleons ($r \approx 10^{-13}$ cm) to relatively huge Rydberg atoms that are a billion times larger (up to $r \approx 10^{-4}$ cm). Similarly, stellar scale objects have a size range of about a billion, from compact neutron stars ($r < 10^6$ cm) to stellar systems with radii of $\leq 10^{15}$ cm.

4. Binary spiral galaxy systems tend to avoid having parallel spins (Helou, 1984) and this phenomenon is also common to atomic scale systems. Additionally, Tifft (1982) has presented data that are suggestive of quantization in the orbital motions of binary galaxies.

5. As will be discussed in more detail in Section 3, both atomic and stellar scale systems tend to have relationships between their angular momenta (J) and masses (M) of the form $J = kM^2$, where k is a constant. Systems on both scales also tend to have relationships between their magnetic dipole moments (μ) and angular momenta of the form $\mu = \Delta J$, where Δ is a constant.

6. The remarkable potential for analogies between the solar system and an atom in a highly excited state (Rydberg atom) has been known for some time (Metcalf, 1980); in general the morphologies, kinematics, and dynamics of Rydberg atoms and their stellar scale analogs are intriguingly similar (Oldershaw, 1982, 1986*a*, 1987*b*). For example, in both cases their radii and oscillation periods are related by laws of the form of Kepler's third law, $P^2 \approx KR^3$, where K is a constant (Oldershaw, 1989*b*).

Potential analogies such as the six listed above, and others discussed in the SSCM references cited above, emboldened me to consider the speculative hypothesis that atomic, stellar, and galactic scale systems might be rigorously self-similar, i.e., that specific systems on a given cosmological scale have specific analogs on all other cosmological scales, and that the properties of analogs from different scales are quantitatively related by simple scale transformation equations.

The derivation of a set of self-similar scale transformation equations, which can relate corresponding length, time, and mass values for analog systems on different scales, was perhaps the most important step toward quantitative testing of the cosmological self-similarity hypothesis, since these equations would allow one to identify analogs on different scales, to assess quantitatively their self-similarity, and to make definitive predictions. From Mandelbrot's (1982) basic discussion of self-similarity, a little physics (e.g., velocities should be scale invariant), and a knowledge of the above-mentioned general properties of the cosmological hierarchy, one can infer that the simplest scaling equations for a highly stratified self-similar hierarchy would be

$$R_N = \Lambda R_{N-1} \tag{1}$$

$$T_N = \Lambda T_{N-1} \tag{2}$$

and

$$M_N = \Lambda^D M_{N-1} \tag{3}$$

where R, T, and M are length, time, and mass values pertaining to analog systems on neighboring cosmological scales N and $N-1$, and where Λ and D are scaling constants that must, for the present, be determined empirically. The values of Λ and D are found to be approximately 5.2×10^{17} and 3.174, respectively, and the methods by which these values were arrived at are discussed in Oldershaw (1986*a*) and in the sequel (Oldershaw, 1989*c*) to this paper. In general, these methods involve identifying a pair of putative analogs for which there are reasonably accurate mass and radius estimates and for which the analogy seems dependable. Ratios of analogous mass and radius measurements then yield Λ and D, since $R_N/R_{N-1} = \Lambda$ and $M_N/M_{N-1} = \Lambda^D$. The analog pair that was initially used consisted of the solar system, for which accurate data are available, and a very highly excited Rydberg atom ($n \approx 168$), an atomic scale system whose basic properties are both quantifiable and strongly analogous to those of the solar system. The fact that Λ and D are single valued rather than multivalued or continuous reflects the fact that according to the SSCM, nature's hierarchy is modeled as having discrete and symmetric stratification.

2.4. Summary of the Basic Model

The SSCM views nature as a highly stratified, nested, and possibly unbounded hierarchy of systems with atomic, stellar, and galactic scale systems comprising a discrete, symmetric framework for the observable portion of the entire quasicontinuous hierarchy. It is further hypothesized that the hierarchy is rigorously self-similar such that radii, periods, masses, and in fact any corresponding parameters (Oldershaw, 1986*a*–*e*, 1987*a*) associated with analog systems on different scales are correlated by the very simple scale transformations defined in equations (1)–(3). Given the currently accepted *theoretical* models of atomic, stellar, and galactic systems, one might be highly inclined to regard the latter hypothesis as being simply impossible, i.e., of having no chance of applying to the real world. So much more surprising, then, will be the results presented below of actual empirical tests of the hypothesis. Nature, rather than human theoretical constructs, should be the template upon which we decide the merits or shortcomings of a scientific hypothesis.

3. EMPIRICAL TESTS OF THE SELF-SIMILAR COSMOLOGICAL MODEL

3.1. Introductory Notes

Table I presents the results of 20 retrodictive falsification tests of the SSCM. As opposed to definitive predictions (Oldershaw, 1988), which predict unexpected phenomena or the results of empirical experiments *before they are known*, retrodictive falsification tests determine a theory's ability to "retrodict" previously known data, i.e., they test a theory's consistency with observations. Therefore, retrodictive falsification tests are inherently less stringent than are tests involving definitive predictions. However, to the extent that a theory can pass a large and diverse array of retrodictive falsification tests, our confidence in the theory as a good approximation to natural phenomena is commensurately increased. The final three tests listed in Table I come reasonably close to being classified as true predictions, since they involve relationships that were not thoroughly characterized prior to the tests; several predictions of the SSCM that unquestionably meet the criteria for definitive predictions have been presented before (Oldershaw 1986*a,d*, 1987*a*) and will be discussed further in a forthcoming paper (Oldershaw, 1989*c*).

Below I will review each test and its results, referencing previous discussions of the test and identifying new data that are applicable. Since all measurements involve uncertainties, the reference, "predicted," and empirical values listed in Table I are estimates and each should be thought of as being preceded by an "approximately equals" symbol. Relevant sources and degrees of uncertainty are discussed in the cited references and in this paper.

An important caveat, already mentioned in Section 2, is that nature does not present us with equivalent samples of atomic, stellar, and galactic scale systems. In terms of numbers of systems and "sampling ratios" (see point 5 of Section 2.2), the values for the atomic, stellar, and galactic scales are about 10^{80}, 10^{23}, and 10^{11}, and 10^{40}, 10^{17}, and 10^{5}, respectively. To put this into bold perspective, what we observe of the galactic scale (Oldershaw, 1986*d*) is analogous to studying the atomic scale on the basis of observing a mere 10^{11} subatomic particles crammed into a volume roughly comparable to that of a single hydrogen atom. This sample would woefully underrepresent the richness of atomic scale phenomena, and therefore we must bear in mind that the available galactic scale sample is similarly limited. The situation is quite a bit better on the stellar scale, but the caveat against assuming equivalent samples is still very important when making stellar-atomic comparisons.

The empirical tests listed in Table I usually have the following format: a reference parameter that has been measured with reasonable accuracy is identified for a class of systems on a given scale, this value is then transformed according equations (1)-(3) in order to yield a "predicted" counterpart value for the analogous class of systems on a different scale, and finally the "predicted" value is compared with empirical measurements made on the relevant class of analog systems. Usually atomic scale systems are chosen as the source of reference parameters because our empirical measurements of atomic scale parameters are in general vastly superior to our quantification of stellar or galactic scale parameters.

3.2. Discussion of Individual Tests

1, 2. Since these two tests are intimately related, it will be convenient to discuss them together. It has been firmly established (Trimble, 1975) that the measured abundances (by numbers rather than by mass) of hydrogen and helium are remarkably constant over a wide variety of cosmologically representative samples: the sun's outer layers, the interstellar medium, meteorites, distant stars, cosmic rays, and other galaxies. Hydrogen appears to account for $90 \pm 2\%$ of all atomic species, helium appears to make up $9 \pm 2\%$ of all atomic species, and elements heavier than helium only contribute about 1% to the total. Given these atomic scale abundance values, the SSCM predicts that comparably representative samples of large numbers of stellar scale systems will reveal that stellar scale hydrogen and helium analogs account for approximately 90% and 9% of the stellar scale sample, respectively. There is a technical difficulty with a straightforward application of this test, but fortunately there is a way to circumvent the problem. The

Table I. Data for Falsification Tests of the SSCM

Test #	Test parameter	Reference parameter	Reference parameter value	Dimensions, scaling factor	SSCM "prediction"	Observed result
1	M dwarf abundance	Hydrogen abundance	$90 \pm 2\%$	—	$\langle \approx 90\% \rangle$	$\langle \approx 89\% \rangle$
2	K dwarf abundance	Helium abundance	$9 \pm 2\%$	—	$\langle \approx 9\% \rangle$	$\langle \approx 10\% \rangle$
3	Lower limit radius For M dwarfs	Lower limit radius for hydrogen	1.6×10^{-8} cm	L, Λ	8.3×10^{9} cm	8.7×10^{9} cm
4	Average radius for white dwarfs	Radius for He^{+}	2.1×10^{-9} cm	L, Λ	1.1×10^{9} cm	0.9×10^{9} cm
5	Lower limit radius for white dwarfs	Lower limit radius for atomic ions	4.2×10^{-10} to 1.2×10^{-9} cm	L, Λ	2.2×10^{8} to 6.1×10^{8} cm	5.5×10^{8} cm
6	Range of radii for MS, G, and SG stars	Range of radii for neutral atoms	1.6×10^{-8} to 6.4×10^{-5} cm	L, Λ	8.3×10^{9} to 3.3×10^{13} cm	8.7×10^{9} to 3.4×10^{13} cm
7	Average mass for white dwarfs	Mass of ^{4}He	6.7×10^{-24} g	M, Λ^{D}	1.14×10^{33} g	1.15×10^{33} g
8	Lower mass limit for white dwarfs	$^{3}He/^{4}He$ mass ratio	0.75	—	8.7×10^{32} g	8.8×10^{32} g
9	Radius of proton	Schwarzschild radius of black hole	$G = 6.68 \times 10^{-8}$ $cm^{3}\, g^{-1}\, sec^{-2}$	$L^{3}/M\,T^{2}$, $\Lambda^{D}\Lambda^{2}/\Lambda^{3}$	0.81×10^{-13} cm	0.8×10^{-13} cm
10	Log of stellar to atomic ratio of K values from $J = KM^{2}$	—	—	$L^{2}/M\,T$, $\Lambda^{2}/\Lambda^{D}\Lambda$	-38.51	-38.41 (± 3.50)
11	Log of stellar to atomic ratio of Δ values from $\mu = \Delta J$	—	—	$L^{0.5}/M^{0.5}$, $\Lambda^{0.5}/\Lambda^{1.59}$	-19.31	-20.36 (± 2.43)

12	Spin period for typical pulsar	Spin period for typical nucleus	5×10^{-20} sec	T, Λ	0.03 sec	0.002–3.0 sec
13	Radius range for galaxies	Radius range for atomic nuclei	0.8×10^{-13} to 8.3×10^{-13} cm	L, Λ^2	2.2×10^{22} to 2.2×10^{23} cm	0.9×10^{22} to 3.1×10^{23} cm
14	Spin period for typical galaxy	Spin period for typical nucleus	5×10^{-20} sec	T, Λ^2	4.3×10^8 years	$4.4 \pm 2.2 \times 10^8$ years
15	Magnetic dipole moment range for neutron stars	Magnetic dipole moment range for atomic nuclei	4.5×10^{-25} to 1.8×10^{-23} G cm^3	$M^{0.5} L^{1.5}$, $\Lambda^{1.59}\Lambda^{1.5}$	$10^{30.34}$ to $10^{31.94}$ G cm^3	$10^{30.3}$ to $10^{31.3}$ G cm^3
16	Preferred oscillation periods for He^+	Preferred oscillation periods for white dwarfs	250–850 sec	T, Λ^{-1}	4.8×10^{-16} to 1.6×10^{-15} sec	5.5×10^{-16} to 1.6×10^{-15} sec
17	Range of vibration periods for neutron stars	Range of vibration periods for atomic nuclei	1.3×10^{-22} to 7.8×10^{-21} sec	T, Λ	6.8×10^{-5} to 4.1×10^{-3} sec	10.0×10^{-5} to 1.2×10^{-3} sec
18	Form of period–radius laws for variable stars	Form of period–radius laws for Rydberg atoms	$p^2 \approx k_i r^3$	—	$P^2 \approx K_i R^3$	$P^2 \approx K_i R^3$
19	Values of K_1 and K_2 from #18	Values of k_1 and k_2 from #18	1.6×10^{-7}, 2.0×10^{-8} sec^2/cm^3	T^2/L^3, Λ^2/Λ^3	3.0×10^{-25}, 3.8×10^{-26} sec^2/cm^3	$\approx 3.0 \times 10^{-25}$, $\approx 3.8 \times 10^{-26}$ sec^2/cm^3
20	Range and distribution of periods for He, $n=7$ to $n=9$	Range and distribution of periods for RR Lyrae stars	0.2–0.8 days, peaks at about 0.32, 0.37, 0.40, 0.44, 0.47, and 0.52 days	T, $(\Lambda^{-1})(\Lambda)$	0.2–0.8 "days", peaks at about 0.32, 0.37, 0.40, 0.44, 0.47, and 0.52 "days"	0.28–0.75 "days", peaks at about 0.32, 0.38, 0.40, 0.44, 0.47, and 0.52 "days"

atomic H and He abundances refer to *total* H and *total* He; this means that atoms in neutral, partially ionized, and fully ionized states are included in the atomic scale abundance determinations. According to the SSCM, our present observational capabilities are not sufficient to detect reliably stellar scale analogs to fully ionized atoms, i.e., bare nuclei, and therefore fully ionized species must be excluded from the comparison (Oldershaw, 1986*c*). If, instead of using total abundances, the abundances of just *neutral* species are chosen for comparison, then on the atomic scale the reference parameter values are essentally the same as for the total abundances, and on the stellar scale all relevant counterparts are observable. By excluding the partially ionzed species from the comparison, the serious complication posed by widely differing ionization potentials is largely avoided.

The SSCM proposes (Oldershaw, 1986*a*) that stars with radii greater than about 9×10^9 cm, e.g., main sequence, giant, and supergiant stars, are stellar scale counterparts to atoms in excited, but for the most part neutral, states. Equations (1)-(3) predict that the stellar scale hydrogen analog has a mass of about $0.15 M_\odot$ and the helium analog has a mass that is four times larger, or about $0.6 M_\odot$. As anticipated by the SSCM, recent data (Lupton *et al.*, 1987; Low, 1985) show a distinct abundance peak at about $0.62 M_\odot$ and a much larger peak that falls somewhere between $0.1 M_\odot$ and $0.2 M_\odot$. Since there is a considerable amount of uncertainty involved in estimating stellar masses [note the broadness of the abundance peaks of Lupton *et al.* (1987)], the stellar scale abundances of H and He analogs will be defined here as the abundances of stars with masses estimated to be in the ranges $0.1 M_\odot$ to $0.4 M_\odot$ and $0.45 M_\odot$ to $0.75 M_\odot$, respectively. These mass ranges correspond quite well to the estimated mass ranges of M dwarf and K dwarf stars, and therefore the SSCM predicts that the abundances of M dwarf and K dwarf stars should be about 90% and 9%, respectively. Quantitative determinations of these stellar abundances are exceedingly hard to find in the literature, but Wood (1966) has made a comprehensive attempt and for his most reliable sample of galaxies the M dwarf abundance ranges from 81% to 95% with an average of 89%, while the K dwarf abundance ranges from 6% to 18% with an average value of 10%. The average values are quite close to the predicted values.

3. Since M dwarf stars are identified with stellar scale analogs to hydrogen in neutral but usually excited states, one can take the ground-state radius for H, scale it according to equation (1), and arrive at a SSCM prediction for the lower limit radius of an M dwarf star. The only difficulty here is that neither an atom nor a star has a distinct boundary at a fixed radius, but rather both have somewhat ephemeral boundaries. The radius encompassing 90% of the electronic charge distribution was chosen (Oldershaw, 1986*a*) as an appropriate estimate for the ground-state radus

of H, and the consequent prediction for the lower limit radii of M dwarf stars (8.3×10^9 cm) was found to be in good agreement with observational estimates of approximately 8.7×10^9 cm.

4. Their masses, abundances, and probable origins in planetary nebulae all serve to identify (Oldershaw, 1986*a,c*) the overwhelming majority of white dwarf stars as stellar scale analogs to He^+ ions in their ground states. Therefore the estimated radius for a ground-state He^+ ion (roughly $0.4a_0$, where a_0 is the Bohr radius) can be scaled according to equation (1) to yield a predicted average radius for white dwarf stars. The resulting prediction of roughly 1.1×10^9 cm is found to be in reasonable agreement with the observationally estimated average radius (Greenstein, 1985) of 0.9×10^9 cm for white dwarf stars, given the uncertainties associated with the latter value. Parenthetically, it has been noted (Oldershaw, 1982) that the morphologies of the structures being ejected in planetary nebula systems, the cores of which are interpreted as predominantly He^+ analogs, are intriguingly similar to the morphologies of electronic wave functions in atoms.

5. Although nearly all white dwarf stars have been identified as He^+ analogs, with masses of approximately $0.45M_\odot$ (see test 8 below) and $0.60M_\odot$, very small numbers of stellar scale analogs to more massive ions are expected to be found in this class of objects. It should be clarified that according to the SSCM the class of white dwarf stars is analogous to the class of highly (but not fully) ionized atomic scale ions with remaining electrons populating very low energy levels. Therefore, if one scales the lower limit radius for atomic ions with masses greater than four atomic mass units according to equation (1), then one should arrive at an SSCM prediction for the lower limit radius for a white dwarf star (Oldershaw, 1986*a*). Ionic radii (Weast, 1971-1972) for almost fully ionized ions more massive than He have a lower limit value of roughly $0.08a_0$, where a_0 is the Bohr radius, and singly ionized ions have a lower limit radius of about $0.22a_0$. It is not entirely clear which lower limit represents the better reference parameter for this test, and so we will only expect that the observed lower limit radius for white dwarf stars (or better, for those whose radii have been estimated so far) will be in the range $(\Lambda)(0.08a_0)$ to $(\Lambda)(0.22a_0)$, or 2.2×10^8 cm to 6.1×10^8 cm. This is found to be the case; the observed lower limit is about 5.5×10^8 cm (Greenstein, 1985).

6. As noted above, the SSCM identifies main sequence, giant, and supergiant stars as stellar scale analogs to excited, but primarily neutral, atoms. The latter have a large range of radii extending from approximately $3a_0$ for the ground state of H to an approximate radius of $12{,}100a_0$ for the largest Rydberg atoms that are commonly observed (Percival, 1980). A coarse but useful test of the SSCM can be achieved by using equation (1)

to scale the limits of this range up to stellar scale values, and to inquire whether this predicted range corresponds to the observed radius range for "normal" stars. Results of this test are in good agreement with expectations of the SSCM, since the lower limit radius for M dwarf stars is roughly $3A_0$, where A_0 is the stellar scale equivalent to the Bohr radius, and supergiant stars have observed radii up to roughly $12{,}140A_0$ (de Vaucouleurs, 1970).

7. Having determined that the stellar scale equivalent to the mass of the hydrogen atom is approximately $0.15M_\odot$ (Oldershaw, 1986*a*), the SSCM predicts that the stellar scale analogs to helium, i.e., K dwarfs and most white dwarfs, will be about four times more massive, or $0.60M_\odot$. Observational results confirm that $0.6M_\odot$ is an excellent estimate for the average mass of K dwarfs (Lupton *et al.*, 1987), and the distribution of masses for white dwarf stars is a surprisingly narrow peak centered on about $0.6M_\odot$ (Schoenberner, 1981; Mallik, 1985). The nuclei of planetary nebulae, which are also identified as He^+ analogs and precursors of white dwarf stars, have a remarkably sharp mass distribution centered on $0.58M_\odot$ (Schoenberner, 1981). The sequel (Oldershaw, 1989*c*) to this paper will contain a discussion of the SSCM prediction that the actual distribution of stellar masses is much more discrete than is inferred at present; the very sharp mass distribution for the nuclei of planetary nebulae is encouraging evidence along these lines.

8. If white dwarf stars are predominantly analogs to helium ions, then they must be primarily analogs to $^4He^+$ ions, which are by far the most common isotope of helium. However, one would expect very small numbers of $^3He^+$ analogs to be included in the present sample of white dwarf stars, and therefore one would predict that the lower limit mass for white dwarf stars is approximately $(3/4)(0.58M_\odot) = 0.44M_\odot$. Two observational estimates (Mallik, 1985; Greenstein, 1985) of this parameter are $0.45M_\odot$ and $0.44M_\odot$.

9. Using equations (1) and (3), one may calculate (Oldershaw, 1986*a*) that the mass and radius of the stellar scale analog to the proton are approximately $0.145M_\odot$ and 0.42×10^5 cm, respectively. It is immediately noticed that the radius for the stellar scale proton analog is very close to the Schwarschild radius (0.428×10^5 cm) for a stellar object with a mass equal to $0.145M_\odot$. According to the SSCM, therefore, the radius of the proton should be equal to the Schwarschild radius for an object with a mass equal to 1.67×10^{-24} g, if the "constants" in the Schwarschild radius equation

$$R_s = 2G_N M / c^2 \tag{4}$$

are scaled to their proper atomic scale values (Oldershaw, 1986*a,d,e*). The value for the velocity of light c is invariant with respect to scale transformations, but the Newtonian gravitational constant G has dimensions $L^3/M\,T^2$ and therefore according to equations (1)-(3) the atomic scale value G_{-1} is $\Lambda^{2.174}$ times larger than the stellar scale value G_0. Solving equation (4) with $M = 1.67 \times 10^{-24}$ g and $G_{-1} = (\Lambda^{2.174})(6.68 \times 10^{-8}\ \mathrm{cm^3/g\,sec^2})$ gives a predicted radius of 0.81×10^{-13} cm; the empirically estimated radius for the charge distribution of the proton (Bethe and Salpeter, 1957) is approximately 0.8×10^{-13} cm.

10, 11. These two tests have been presented in detail before (Oldershaw, 1986*b*) and here I will only outline the rationale for the tests and repeat the results. It has been observed that many stellar scale systems have a relationship between their masses M and angular momenta J of the form $J = K_s M^2$, where K_s is a constant with dimensions $L^2/M\,T$. Similarly, it has been observed that families of atomic scale systems obey relationships of the form $J = K_a M^2$. According to the scaling rules of the SSCM, the logarithm of K_s/K_a should have a value of approximately -38.51, and this is in good agreement with the rough empirical estimate of -38.41 (± 3.50). Likewise, both stellar scale and atomic scale systems tend to have a relationship between their magnetic dipole moments μ and angular momenta J of the form $\mu = \Delta J$. The scaling rules of the SSCM lead to the expectation that the logarithm of Δ_s/Δ_a should be about -19.31, which can be compared with the empirical estimate of -20.36 (± 2.43). Because of the very large error bars on the empirical values for K_s/K_a and Δ_s/Δ_a, these tests only show that the SSCM predictions for K_s/K_a and Δ_s/Δ_a are "in the right ballpark." Perhaps in the future improved empirical constraints will permit more stringent versions of these tests.

12. A classical spin period for a typical atomic scale nucleus is estimated to be about 5×10^{-20} sec (Oldershaw, 1986*d*). Since the SSCM identifies neutron stars as stellar scale analogs to atomic nuclei, equation (2) can be used to scale up the nuclear spin period of 5×10^{-20} sec to an expected value of approximately 0.03 sec for the spin period of a typical neutron star. To date, the observed range of spin periods for pulsars is 0.002 sec to about 3.0 sec, and therefore the predicted spin period does fall within the empirical range for neutron stars. On the other hand, it should be mentioned that pulsar spin periods in the range 0.1 to 1.0 sec are far more common in present samples than those below 0.1 sec. An intriguing phenomenon that both atomic nuclei and pulsars share is that of abrupt "glitches" (Stephens, 1985) wherein the spin frequency of the system seemingly instantaneously goes from regular decrease to a significantly higher value and then resumes a slow decrease from the higher frequency. The observed (Lyne, 1987) pulsar "glitches" have so far involved only very small frequency jumps ($\Delta f/f \leq 10^{-6}$) as compared with the very large "glitches" ($\Delta f/f$ on the order of 10^{-1}) seen in atomic nuclei, but the analogy is an interesting one and

perhaps comparably large pulsar "glitches" will be observed in the future. To date, only 14 "glitches" in 7 pulsars have been observed, but statistics indicate that they should be a very common phenomenon.

13. If equations (1)-(3) do relate self-similar phenomena on different scales of the cosmological hierarchy, then the high-velocity (average value $\geq$400 km/sec) random motions of galaxies unambiguously require that their analogs on the atomic scale are atomic nuclei under fully ionized plasma conditions (Oldershaw, 1986*d*). Therefore, if the range of radii for atomic nuclei, which is about 0.8×10^{-13} to 8.3×10^{-13} cm, is scaled up to galactic scale values according to equation (1), i.e., multiplied by Λ^2, then the resulting range of about 2.2×10^{22} to 2.2×10^{23} cm should compare favorably with the empirically estimated range for the radii of galaxies. There is a significant amount of uncertainty in galactic radius estimates, primarily because the Hubble constant is uncertain by a factor of 2 and the exact extent of dark matter haloes of galaxies is often difficult to estimate. However, the smallest galaxy for which the dark matter halo has been taken into account (Kormendy, 1985) has a radius of $\geq 0.9 \times 10^{22}$ cm and the largest galaxies (Saslaw, 1985) have radii of roughly 3.1×10^{22} cm. The agreement between the predicted and the empirically estimated radius ranges is quite good considering the present observational uncertainties, and a more exact correspondence is a viable possibility. Both galaxies and atomic nuclei have shapes that are well represented by McClaurin spheroids and Jacobi ellipsoids, including prolate and triaxial shapes (Oldershaw, 1986*d*).

14. In test 12 a typical spin period for an atomic nucleus and the range of spin periods for pulsars were shown to be correlated in a manner that was consistent with the SSCM predictions. A further SSCM prediction is that multiplying the atomic scale spin period of about 5×10^{-20} sec by Λ^2, in accordance with equation (2), should yield a typical galactic spin period. The numerical value is about 4.3×10^8 years, and this spin period is approximately equal to a rough estimate (Mihalas and Binney, 1981) of the spin period of 4.4 $(\pm 2.2) \times 10^8$ years for our galaxy, which is in all respects a typical galaxy.

15. As mentioned above, the SSCM unambiguously identifies atomic scale nuclei and stellar scale neutron stars as self-similar analogs. Therefore the range of magnetic dipole moments μ, with dimensions of $M^{1/2} L^{3/2}$, for atomic nuclei should be related to the μ range for neutron stars by a scaling factor of $(\Lambda^{D/2})(\Lambda^{3/2}) = 4.9 \times 10^{54}$. The range of μ values for cosmologically abundant atomic nuclei (Oldershaw, 1987*a*) is 4.5×10^{-25} to 1.8×10^{-23} G cm^3, and so the predicted range for neutron stars is roughly $10^{30.34}$ to $10^{31.94}$ G cm^3. The estimated range of μ values for neutron stars is roughly $10^{30.3}$ to $10^{31.3}$ G cm^3 (Kundt, 1986), which is in good agreement with the predicted range, given the theoretical and empirical uncertainties involved in this test.

16. Since the majority of white dwarf stars have been identified as self-similar analogs to He^+ ions, and since white dwarfs appear to have preferred oscillation periods (Wesemael *et al.*, 1986) of approximately 250 ($\pm$100) and 850 ($\pm$100) sec, it can be predicted via equation (2) that He^+ ions should have major transition periods of about $(250\,\text{sec})(\Lambda^{-1}) = 4.8\times 10^{-16}$ sec and $(850\,\text{sec})(\Lambda^{-1}) = 1.6\times 10^{-15}$ sec. In fact, these predicted periods are in good agreement with two of the three major transiton periods of He^+ ions: 5.5×10^{-16} and 1.6×10^{-15} sec (Oldershaw, 1989*a*).

17. Since neutron stars have been identified as self-similar counterparts to atomic scale nuclei (Oldershaw, 1986*a*), the SSCM predicts that the ranges of vibrational periods for these two classes of systems should be correlated by equation (2). Vibration periods in atomic nuclei range from about 1.3×10^{-22} to 7.8×10^{-21} sec, and therefore the anticipated range of vibration periods for neutron stars should be approximately 6.8×10^{-5} to 4.1×10^{-3} sec (Oldershaw, 1989*a*). This predicted range is in good agreement with the empirically determined range of 10.0×10^{-5} to 1.2×10^{-3} sec considering that the latter range is based on a very small sample size (Carroll *et al.*, 1986).

18, 19. The SSCM identifies the majority of main sequence, giant, and supergiant stars as stellar scale analogs to neutral atoms in highly excited Rydberg states (Oldershaw, 1986*a*, 1987*b*). It also predicts that any well-defined physical phenomenon observed on either the atomic or stellar scale will have an analogous counterpart on the other scale. When Rydberg atoms undergo transitions to lower energy states they oscillate with periods p that are related to the average radii r in the following manner (Percival, 1980):

$$p^2 \approx k_1 r^3 \qquad (\text{for } l \approx n) \tag{5}$$

and

$$p^2 \approx k_2 r^3 \qquad (\text{for } l \ll n) \tag{6}$$

where n is the principal quantum number, l is the azimuthal quantum number, k_1 is a constant equal to $(p_0)^2/(a_0)^3$, and k_2 is a constant equal to $(p_0)^2/(2a_0)^3$. The parameter p_0 is the minimum transition period for hydrogen and a_0 is the Bohr radius. From the fact that Rydberg atoms obey approximate relationships of the form of Kepler's third law, i.e., $p^2 \approx k_i r^3$, it can be predicted that variable stars with radii $\geq 1 R_\odot$, which have been identified as stellar scale analogs to Rydberg atoms undergoing transitions to lower energy states (Oldershaw, 1987*b*), will have periods P and radii R that obey approximate relationships of the form $P^2 \approx K_i R^3$, where the K_i represent analogs to the k_i. It has been demonstrated (Oldershaw, 1989*b*) that a wide variety of variable stars, including delta Scuti, RR Lyrae, beta

Cepheid, classical Cepheid, and supergiant variables, do indeed obey period–radius relations of the predicted form. Moreover, it has been shown that the atomic scale constants k_1 and k_2 are quantitatively related to their stellar scale counterparts K_1 and K_2 by the self-similar scaling rules embodied in equations (1)–(3). A third $P-R$ relationship with a K_3 value that is closely related to K_1 and K_2 has been identified for variable stars and has led to the prediction that an analogous p–r relation will be found for a subset of Rydberg atoms (Oldershaw, 1989*b*).

20. This final test of the SSCM requires more discussion than its predecessors because it has not been published previously. The proposed analogy between variable stars and Rydberg atoms undergoing transitions leads to the expectation that the periods of variable stars have quantized values, as is the case with their atomic scale analogs. This expectation will be explored below, but several important caveats must be mentioned first. When atomic scale quantization is observed, one of two general strategies has been employed: strategy A is to observe a perfectly homogeneous sample of atoms under rigorously controlled ambient conditions, and when strategy A is not feasible because one cannot regulate the homogeneity of the atomic species and/or the ambient physical conditions, then strategy B is to sample enormous numbers ($\geq 10^{20}$) of atoms in the hope that discrete peaks will rise above the nearly continuous background. On the stellar scale one is faced with the following observational circumstances.

(a) Variable stars represent a heterogeneous mixture of stellar scale counterparts to atoms and ions.
(b) Values of n for individual stars can range from 1 to at least 100.
(c) For *each* value of n there are n different energy levels due to orbital angular momentum considerations, i.e., l can vary from 0 to $n-1$ for each value of n.
(d) If spin considerations are included, then the above-mentioned energy levels are further split into an even larger set of levels.
(e) Since the energy levels of Rydberg atoms can be significantly shifted by ambient electric and magnetic fields, the SSCM asserts that an analogous shifting of energy levels can also occur in the case of their stellar scale counterparts. Variable stars from different locations within our galaxy, i.e., near the nucleus, in the outer halo, in the spiral arms, or in globular clusters, would therefore be expected to have period distributions that are influenced by differing *galactic scale* electromagnetic environments.

If careful thought is given to these five considerations, which would serve to generate a dense "forest" of transition periods for Rydberg atoms or their analogs, then it is clear that expecting to find textbook-style evidence for quantized periods among variable stars, based on a maximum sample size on the order of 10^4 periods, is essentially ruled out at present. Under the existing observational circumstances even less overt evidence for quantization in atoms or variable stars would still be very difficult to obtain, since strategy B is precluded by having only a tiny sample of systems and since strategy A is hampered by our inability to manipulate the sample or the ambient physical conditions that affect the sample. However, all is not lost. Granted that blatant examples of quantization are not to be expected, one might still hope to observe less overt evidence of quantization in the following manner. Since the sample size is invariably going to be small, the best strategy is to identify as homogeneous a subsample of variable stars as possible, with the hope of minimizing the number of different species, the spread of n and l values, and the influence of differing ambient physical conditions.

RR Lyrae stars constitute perhaps the best candidate for a class of variable stars that meets the desired criteria. Their masses are found to cluster around $0.6M_{\odot}$ (Stothers, 1981) and therefore the SSCM unambiguously identifies them as primarily helium analogs. The overwelming majority of their radii fall within the range of $4R_{\odot}$ to $7R_{\odot}$ (Stothers, 1981), and from this fact the SSCM identifies (Oldershaw, 1987*b*) the range of n values for RR Lyrae variables as $n=7$ to $n=9$. Also, their position on a period-radius graph shows that they represent the $l \ll n$ case (Oldershaw, 1989*b*); here we will assume that $l \leq 2$. Therefore, if reasonably large samples of RR Lyrae variables from reasonably homogeneous galactic environments are analyzed in terms of relative frequencies of oscillation periods, then the SSCM anticipates that evidence for discrete, preferred periods will be present, though the statistical significance might be low. The range of the period distribution and the preferred periods for the RR Lyrae stars should be correlated with corresponding He transition periods in a manner consistent with equation (2).

I have investigated the period distributions for several RR Lyrae subsamples taken from the *General Catalogue of Variable Stars* [the Third Edition and its Supplements] (Kukarkin *et al.*, 1969–1970), and two useful empirical findings have resulted from these investigations. First, about 99% of the RR Lyrae stars have periods in the range 0.2–0.8 day. Second, there tend to be recurrent peaks in the subsample period distributions at periods of 0.32 ± 0.01, 0.37 ± 0.01, 0.40 ± 0.01, 0.44 ± 0.01, 0.47 ± 0.01, and 0.52 ± 0.01 day. The strengths of these preferred periods varies from subsample to subsample and their position is sometimes shifted by ± 0.01 day, but their recurrence in different subsamples lends credibility to the hypothesis that RR Lyrae variables have preferred periods. As an example, Table II presents the distribution of periods for a subsample of 672 RR Lyrae variables that

Table II. Distribution of Periods for a Sample of 672 RR Lyrae Variables and Relevant Scaled Transition Periods for H, He, and Li Atoms

RR Lyrae period distribution				Scaled atomic periods		
ΔP (days)	N	ΔP (days)	N	H	He	Li
0.150–0.159	1	0.500–0.509	15	0.383	0.276	0.250
0.160–0.169	0	0.510–0.519	32	0.558	0.287	0.256
0.170–0.179	2	0.520–0.529	→35		0.323	0.362
0.180–0.189	1	0.530–0.539	35		0.326	0.369
0.190–0.199	0	0.540–0.549	30		0.354	0.378
0.200–0.209	1	0.550–0.559	23		0.378	0.398
0.210–0.219	0	0.560–0.569	27		0.389	0.529
0.220–0.229	2	0.570–0.579	22		0.404	0.543
0.230–0.239	4	0.580–0.589	18		0.406	0.579
0.240–0.249	2	0.590–0.599	13		0.424	0.590
0.250–0.259	2	0.600–0.609	14		0.432	0.798
0.260–0.269	5	0.610–0.619	9		0.440	0.866
0.270–0.279	5	0.620–0.629	7		0.464	
0.280–0.289	5	0.630–0.639	11		0.474	
0.290–0.299	7	0.640–0.649	10		0.478	
0.300–0.309	4	0.650–0.659	9		0.513	
0.310–0.319	4	0.660–0.669	4		0.518	
0.320–0.329	→11	0.670–0.679	0		0.552	
0.330–0.339	10	0.680–0.689	2		0.565	
0.340–0.349	8	0.690–0.699	6		0.590	
0.350–0.359	8	0.700–0.709	3		0.633	
0.360–0.369	12	0.710–0.719	3		0.645	
0.370–0.379	→14	0.720–0.729	2		0.681	
0.380–0.389	6	0.730–0.739	0		0.752	
0.390–0.399	5	0.740–0.749	1			
0.400–0.409	→16	0.750–0.759	1			
0.410–0.419	4	0.760–0.769	0			
0.420–0.429	7	0.770–0.779	1			
0.430–0.439	16	0.780–0.789	0			
0.440–0.449	→28	0.790–0.799	1			
0.450–0.459	27	0.800–0.809	2			
0.460–0.469	24	0.810–0.819	0			
0.470–0.479	→43	0.820–0.829	1			
0.480–0.489	25	0.830–0.839	0			
0.490–0.499	26	0.840–0.849	0			

were listed for the Sagittarius region in the *Second Supplement to the Third Edition of the General Catalogue of Variable Stars* (Kukarkin *et al.*, 1974). This typical subsample shows that the range of 0.2–0.8 day includes almost all of the observed periods and it has what appear to be distinct peaks at the six values listed above.

Assuming that RR Lyrae variables correspond to the case $7 \leq n \leq 9$, $l \leq 2$, and $\langle n_2 - n_1 \rangle = 1$, one can compare the stellar scale results with the corresponding transition period data for He, including singlet and triplet systems. For ease of comparison the He periods are scaled up to units of "days," i.e., multiplied by a factor of Λ in accordance with equation (2). The scaled He periods are determined by the calculation

$$P = (h/\Delta E)(\Lambda) \tag{7}$$

where P is the transition period scaled up to "days," ΔE is the energy level separation (Bashkin and Stoner, 1975), Λ is the scaling constant from equations (1)-(3), and h is Planck's constant. The resulting transition period data for He are given in Table II. The range of relevant transition periods for He is 0.276 to 0.752 "days," which is in reasonable agreement with the predicted range of 0.2 to 0.8 "days." And as predicted, the set of 24 transition periods for He contains counterparts to each of the six preferred periods identified in the RR Lyrae sample. In order to test the uniqueness of the correspondence between the He periods and the preferred RR Lyrae periods, the same calculations were undertaken for hydrogen and lithium atoms, and the relevant transition periods for these atoms are also listed in Table II. There is no correlation between the H periods and the preferred RR Lyrae periods. In the case of Li, three of the preferred RR Lyrae periods, including the largest and most discrete peak, have no counterparts among the scaled Li transition periods. Thus, the correspondence between the He and RR Lyrae periods appears to be unique.

This first attempt to investigate the possibility of quantization in the periods of variable stars represents a very approximate test that involves a small sample size and relies on numerous assumptions. Yet the results are reasonably encouraging and they suggest the way to achieve more rigorous quantization tests in the future (Oldershaw, 1989*b*). Such tests would require much larger samples of variable stars that have been segregated according to galactic location, and they would require highly accurate period, radius, and mass data. It would also be interesting to test whether the relative peak heights of the preferred periods for variable stars match up with the transition probabilities for the corresponding periods of the atomic scale analogs.

3.3. Implications of the Empirical Tests

There are only three plausible explanations for the general agreement between the predictions and the empirical data in the 20 tests discussed above: chance, fudging of various types, or cosmological self-similarity. A rough and very conservative calculation of the probability that the agreement could have resulted by chance can be made in the following manner. Assume that the probability of a chance agreement for each test is $\leq 1/3$, i.e., the prediction could be unacceptably high, unacceptably low, or within the error bars of the relevant empirical parameter. Then the *maximum* probability that the same set of scaling rules could pass 20 such tests by chance is $(1/3)^{20}$ or one chance in 3,486,784,424 tries, which is to say that chance would be an extremely unscientific explanation for the favorable results presented in Table I.

Fudging, or arbitrarily adjusting a theory so that it comes into agreement with observational data, has always been and still is (Oldershaw, 1988) a standard tool of the theoretician, though one that tends to be used furtively. The question to be considered here is whether fudging could account for the apparent success of the SSCM and its scale transformation equations. If one could make arbitrary choices with regard to the proposed analog pairs, the form of the scaling equations, and/or the values of the constants appearing in the scaling equations, then could the arbitrarily fudged theory pass the 20 falsification tests presented above even though nature was not fundamentally self-similar? The author's answer to this question, after investigating such matters for over 10 years, is that if self-similarity was not a global property of nature, then a fudged theory that could pass these particular falsification tests, or equally fundamental ones, would be hopelessly complicated and arbitrary. Moreover, as one tried to test the theory beyond the data that it was constructed around, it would quickly fail. In contrast, the SSCM has *very simple* conceptual foundations and scaling equations, the identifications of analog pairs are always based on two or more fundamental properties such as mass, radius, and spin period, and nearly half of the tests (numbers 10–12 and 14–20) were conceived and conducted *after* the theoretical foundations of the SSCM, its major analog pair identities, the form of the scaling equations, and the values of Λ and D had been submitted for publication (Oldershaw, 1986*a*). In the absence of a convincing demonstration to the contrary, for example, a demonstration that an equally simple and successful alternative to the SSCM can be arbitrarily constructed, the fudging explanation is scientifically untenable. The number, diversity, and fundamental nature of the quantitative falsification tests passed by the SSCM strongly support the contention that nature manifests discrete cosmological self-similarity and that equations (1)–(3) uniquely relate the physical properties of atomic, stellar, and galactic scale systems.

4. CONCLUSIONS

In this paper the general concepts and the self-similar scale transformation equations of the SSCM have been discussed, and 20 successful tests have been presented. The simplicity of this model and its ability to relate quantitatively atomic, stellar, and galactic scale phenomena suggest that a new property of nature has been identified: discrete cosmological self-similarity. Although the SSCM is still in the early heuristic stage of development, it may be the initial step toward a truly remarkable unification of our considerable, but fragmented, physical knowledge. Major questions yet to be answered concern the exactness of the cosmological self-similarity (i.e., is the self-similarity accurate only to a factor of about 2 or is it exact) and the number of scales in the cosmological hierarchy (i.e., finite or infinite). It has been argued previously that these two questions are interrelated (Oldershaw, 1981*b*); for example, exact self-similarity necessitates an infinite hierarchy. An even more fundamental question is: why should nature be globally self-similar and rife with examples of local self-similarity?

A forthcoming paper (Oldershaw, 1989*c*) on the SSCM will discuss the paradigm in more technical detail. It will also review several definitive predictions by which the SSCM can be put to very rigorous tests, and it will discuss major unresolved problems that raise doubts about some aspects of the model. The review will conclude with a discussion of the diverse implications of the SSCM.

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